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ROYAL AIRCRAFT ESTABLISHMENT

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ROLLING MOMENT DUE TO SIDESLIP

PART I.

THE EFFECT OF DIHEDRAL

by

I. LEVACIC, Ph.D.

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ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

Rolling moment due to sideslip
Part I. The effect of dihedral - $(\ell_v)_\Gamma$

- by -

I. Levačić, Ph.D.

SUMMARY

This is the first of three reports dealing with the rolling moment derivative due to sideslip. In this report the effect of dihedral is considered; the second report will be concerned with the effect of sweepback, and the third will deal with wing-body interference and the effect of the tail unit. The main outcome of this first part is as follows:

1. The effect of dihedral in sideslip is equivalent to that of a skew-symmetric change in the incidence distribution in unyawed flight, such as is caused by equal up and down movements of the ailerons. The theory of the rolling moment caused by aileron movement is well developed, and is readily transformed to apply to the case of dihedral in sideslip.

2. For wings of elliptic plan form the rolling moment is shown to be proportional to the dihedral and sideslip angles (assumed small), to a function of the dihedral span (Fig. 2) and to a function of the aspect ratio (Fig. 6a). The form of this latter function is the same as that controlling the change (with aspect ratio) of the lift curve slope in straight unyawed flight, except that it relates to an effective aspect ratio equal to half of the geometric aspect ratio.

3. For tapered wings the same conclusions are found to apply, and in fact for rough estimates the effect of taper may be ignored and wings of normal plan form can be treated as elliptical. More accurately, the effect of dihedral span and the variation of the constant of proportionality, i.e. $\ell_{v1/\Gamma}$, with taper can be obtained from Figs. 3 and 4.

4. Comparison of the theoretical deductions with the available experimental results shows very satisfactory agreement (Fig. 4); this comparison indicates that sweepback and interference due to the presence of a fuselage or nacelles have negligible effects on the contribution to ℓ_v due to wing dihedral.

5. The contribution to ℓ_v caused by wing flexure under the aerodynamic loads in flight is shown to be simply expressible in terms of tip

deflection, and, in practice this effect is only slightly dependent on wing taper and can be readily determined for the type of flexure that normally occurs (nearly parabolic); (Figs. 8 and 9).

INDEX
Part I

NOTATION

Dimensions, angles and velocities

s	ft.	Semi-span of the wing
c	ft.	Wing chord at a station along the semi-span
c_o	ft.	Wing chord at the root (see Fig. 1a.)
c_t	ft.	Wing chord at the tip (see Fig. 1a.)
S	sq.ft.	Wing area
$A\!\!R$		Geometric aspect ratio; $A\!\!R = \frac{4\,s^2}{S}$
τ		Wing taper ratio; $\tau = {}^{c_t}/_{c_o}$ (see Fig. 1a)
y		Position along the lateral (Y) axis expressed as a fraction of the semi-span
y_d		Position along the lateral (Y) axis, where the uniform dihedral angle begins, expressed as a fraction of the semi-span; $y_d = 1 - {}^{s_d}/_{s}$
s_d	ft.	Length measured from the tip to the position of the beginning of a uniform dihedral angle; $s_d = s(1 - y_d)$; see Fig. 1b.
${}^{s_d}/_{s}$		Dihedral span ratio; ${}^{s_d}/_{s} = 1 - y_d$, see Fig. 1b.
δ_z	ft.	Flexural deflection at a local station along the semi-span; see Fig. 1c.
δ_{z_1}	ft.	Flexural deflection of the wing tips.
η_s		Distance from rolling axis of lateral centre of pressure due to skew symmetric loading expressed as a fraction of the semi-span.
n		Index of the simple power law of flexural deflection.
α	rad.	Effective geometric wing incidence.
α_n	rad.	Constant geometric incidence of the whole wing.
$\Delta\alpha$	rad.	Part span (or full span) skew symmetric incidence distribution of constant magnitude.
γ	rad.	Local dihedral angle.
Γ	rad.	Part span or full span constant dihedral angle.
Γ_δ	rad.	Dihedral angle corresponding to the same tip deflection as that of a flexed wing; $\Gamma_\delta = \delta_{z_1}/_{s}$ (see Fig. 1c).
β	rad.	Angle of sideslip; positive when the starboard tip is forward.

w	ft./sec.	Velocity component parallel with the z axis.
V	ft./sec.	Resultant wing velocity.
p	sec^{-1}	Angular velocity in roll.
$p.s/V$		Change in the local wing tip incidence due to rate of roll.

<u>Force and moment coefficients, derivatives, components of ℓ_v etc.</u>

C_L		Lift coefficient; $C_L = {}^{Lift}/_{\frac{1}{2}\rho V^2 S}$
c_L		Local lift coefficient.
a_∞		Rate of change of the lift coefficient with incidence for an infinite wing; $a_\infty = \partial C_L/_{\partial\alpha}$
a		The same as above for finite aspect ratio and symmetric distribution of loading.
a_s		Function representing aspect ratio effect on rolling moment of finite span wing due to a skew symmetric loading distribution.
C_ℓ		Rolling moment coefficient; $C_\ell = \dfrac{R.M.}{\frac{1}{2}\rho V^2 S 2s}$
ℓ_v		Rate of change of rolling moment coefficient with angle of sideslip; $\ell_v = \dfrac{\partial C_\ell}{\partial\beta}$, where β is in radians.
ℓ_{v_1}		ℓ_v due to full span dihedral distribution.
		Contributions to ℓ_v due to:-
$(\ell_v)_\gamma$		General dihedral distribution.
$(\ell_v)_\Gamma$		Constant dihedral distribution.
$(\ell_v)_{\delta_s}$	or	tip deflection due to wing flexure.
$(\ell_v)_{\Gamma_\delta}$		
η_γ		Dihedral efficiency.
K		Circulation at local wing section.

1. Introduction

The value of the rolling moment derivative of an aircraft in sideslip is of first importance in assessing its lateral stability characteristics. There can be no doubt, therefore, of the value of a reliable method for predicting ℓ_v in the design stage of an aircraft. Such methods as have been previously developed have been largely empirical, and have not covered all the factors that are known today to be of importance. The approach that has been adopted here is to make the fullest practical use of theory and, by means of the resulting formulae, to interpret the available experimental data. In this way it is believed all the important parameters are adequately considered.

The subject readily divides itself under three main headings; each is considered in a separate report denoted as Parts I, II and III. These three parts are as follows:

Part I	-	The effect of dihedral - $(\ell_v)_\Gamma$
Part II	-	The effect of sweepback.
Part III	-	The effect of wing body arrangement and tail unit.

This first report therefore deals with the effect of dihedral. In paragraphs 2 and 3 elliptic and tapered wings with constant full span dihedral are considered. The effect of uniform part span dihedral for elliptic and tapered wings is considered in paragraphs 3.1. and 3.3. The general case of a variable dihedral along the span is treated in paragraph 4; this includes the case of a wing divided into various spanwise parts each having a different constant dihedral. It also includes the case of dihedral due to wing flexure under the aerodynamic loads in flight. In addition to a summary of the main conclusions the main formulae are collected in paragraph 6.2. and an example is given to illustrate their use.

Acknowledgement

Acknowledgement is due to Mr. A.D. Young for the considerable help given in discussion of the theoretical background to the investigation and in the writing of the report.

2. Effective change of incidence and rolling moment due to full span dihedral in sideslip

It is easily shown (see for example ref.9) that, if a wing with dihedral is put into a sideslip, there is a change in the local effective angle of incidence, which on the starboard side is equal but of opposite sign to the corresponding change on the port side. Such a change in the effective incidence distribution may be conveniently referred to as skew-symmetric. Adopting the usual conventions of sign, a positive angle of sideslip with positive dihedral causes an increase of incidence on the starboard wing and a decrease of incidence on the port wing. This incidence change $(\Delta\alpha)$ is easily shown to be related to the angle of sideslip β and local dihedral angle Γ by the formula

$$\left.\begin{aligned} |\tan(\Delta\alpha)| &= \tan\beta . \sin\Gamma, \\ \text{or } \Delta\alpha &\simeq \beta.\Gamma \text{ ; if } \beta \text{ and } \Gamma \text{ are small} \end{aligned}\right\} \quad \ldots\ldots\ldots 1$$

This skew-symmetrical change of effective incidence will result in equal and opposite changes of lift on the port and starboard wings, and hence in a rolling moment. To arrive at a rough estimate of this rolling moment, the following rather crude assumptions have frequently been made 9, 10, 11.

(a) Changes in induced downwash at the wings due to the skew symmetric changes in effective incidence are negligible.

(b) The rate of change of lift coefficient with incidence for each half wing for the skew symmetric incidence distribution is the same as that for the wing in normal straight unyawed flight.

(c) Any distortion of the trailing vortices and changes in the flow over the tips due to the three dimensional character of oblique flow are neglected.

If these assumptions are applied to the case of a wing of aspect ratio $\mathcal{R}$ with full span constant dihedral Γ, then the lift on each half of the wing, when expressed as a coefficient, is

$$\frac{\beta . \Gamma . s^2}{S} \int_0^1 a \frac{c}{s} \, dy = \frac{a}{2} \beta \Gamma, \qquad \ldots\ldots\ldots\ldots 2$$

where a is the lift curve slope of the wing of aspect ratio $\mathcal{R}$ in straight unyawed flight. If η is the distance of the lateral centre of pressure of this lift from the rolling axis (expressed as a fraction of the semi-span), then the rolling moment coefficient is

$$-C_{\ell} = \frac{a}{2} \eta \beta \Gamma,$$

and hence

$$-\frac{\ell_{v_1}}{\Gamma} = \frac{1}{\Gamma} \frac{\partial C_{\ell}}{\partial \beta} = \frac{a}{2} \eta . \qquad \ldots\ldots\ldots\ldots 3$$

If we take $\eta = 0.5$, and $a = 4.3$ as representative values for a rectangular wing of aspect ratio 6.0, then equation 3 gives

$$-\frac{\ell_{v_1}}{\Gamma} \simeq 1.1.$$

However, the results of experiments summarised in Fig.4 (curve E) show that the actual value of $-\ell_{v_1}/\Gamma$ for a rectangular wing is about 0.8.

Similarly, the estimated value for an elliptic wing (taking $\eta_{ell.} = 4/3\pi$), derived from equation 3, is also about 25% too high when compared with the experimental value for tapered wings approximating the elliptic plan form.

The above crude theory can, however, be readily improved upon; to do this we discard assumptions (a) and (b) above. This is tantamount to assuming that the effect of sideslip on the lift distribution on a wing with dihedral is exactly equivalent to that caused by a skew symmetric change of effective incidence in unyawed flight (such as, for example, is caused by equal and opposite aileron movement). This latter case was considered by Munk[2] for an elliptic wing, and by Hartshorn for elliptic and tapered wings[5]. The theory for an elliptic wing with full span uniform dihedral Γ is reproduced in Appendix I, where it is shown that the rolling moment coefficient due to sideslip is given by

$$\left.\begin{aligned} -\frac{\ell_{v_1}}{\Gamma} &= \frac{a_s}{2}\;\frac{4}{3\pi}, \\ \text{where} \qquad & \\ a_s &= \frac{1}{\frac{1}{a_\infty}+\frac{2}{\pi A}} . \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots 4$$

If we consider, for example, a wing of aspect ratio 6.0 for which $a_\infty = 5.5$, the theoretical value of $-\ell_{v1}/\Gamma$ is now

$$-\frac{\ell_{v_1}}{\Gamma} = 0.738.$$

The mean experimental value corresponding to a taper ratio $\tau \approx 0.35$ is about 0.71 (see Fig.4, curve E). Thus, we find that the value given by equation 4 is only about 5% higher than the experimental value. The tests on tapered wings from which the experimental values shown in Fig.4 have been derived have all been made at small Reynolds numbers. It is not certain whether this may not be responsible for some small reduction of the measured rolling moment due to dihedral in sideslip as compared with what may be expected at full scale Reynolds numbers. Clearly, a scale effect on local lift curve slope will be accompanied by a scale effect on ℓ_v. It appears, therefore, that the theory, based on the flow associated with unyawed flight and the same skew symmetric incidence distribution, can be used with confidence in predicting dihedral effects in sideslip.

If we compare equation 4 with equation 3 it will be seen that we can conveniently regard a_s as an <u>effective</u> lift curve slope for a skew symmetric incidence distribution and $4/3\pi$ as the corresponding value of η_s (or the non-dimensional distance of the lateral centre of pressure from the rolling axis). This view point can be adopted only for the purpose of assessing the rolling moment derivative; in actual fact a_s as given by equation 4 is not the true theoretical lift slope nor is $4/3\pi$ the true theoretical value of η_s. The calculations of Hartshorn[5] show that the <u>true</u> theoretical values of the lift curve slope and the lateral centre of pressure for a full span skew symmetric load distribution on an elliptic wing are approximately

$$\left.\begin{aligned} a_{s(\text{true})} &\doteq \frac{1}{\frac{1}{a_\infty}+\frac{3}{\pi . A}}, \\ \text{and} \qquad & \\ \eta_{s(\text{true})} &\doteq 0.50. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots 5$$

However, a point of very real importance emerges from this comparison. The value of a, the lift curve slope in straight unyawed flight, is given by

$$a = \frac{1}{\frac{1}{a_\infty} + \frac{1}{\pi . R}} \qquad \ldots\ldots\ldots\ldots 6$$

...... The effective value of a_s given by equation 4 is therefore the same as that of a, (cf. equation 6) corresponding to an aspect ratio one half of the true aspect ratio, whilst the true theoretical value of a_s, (as given by equation 5), is the same as that of a for an aspect ratio one third of the true aspect ratio. The true lift curve slope for each half wing for a uniform full span skew-symmetric change of incidence is therefore not the same as the lift curve slope for straight symmetrical unyawed flight (as was assumed in the crude theory[9,10,11]), but corresponds to the lift curve slope when the aspect ratio is reduced to about one third of its true value*. This fact will undoubtedly be of importance in any general analysis of data dealing with other derivatives arising from skew-symmetric loading distributions e.g. ℓ_p and ℓ_r.

3. Rolling moment in sideslip due to uniform part span dihedral

3.1. Elliptic wing

We have seen that in the case of uniform full span dihedral the experimental results agreed very well with the theory based on the assumption that the dihedral in sideslip was equivalent to a full span skew symmetric incidence change in straight unyawed flow. It is reasonable, therefore to extend this theory to the case of uniform part span dihedral, the theory then being identical with that developed by Munk[2] and Hartshorn[5] for part span ailerons. The relevant parts of

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* Rotta has developed a theory for an elliptic wing in sideslip which leads to the following results for the lift curve slope and lateral centre of pressure

$$a_s \approx \frac{1}{\frac{1}{a_\infty} + \frac{2.47}{\pi . R}}$$

and

$$\eta_s = \frac{\pi}{6} = 0.526.$$

This leads to a value of $-\ell_v$ about 18% higher than the experimental value. It may be noted however, that Rotta's theory is based on the assumptions that the downwash is effectively constant and the trailing vortices suffer no distortion. These assumptions lead to reasonable results when applied to symmetric loading problems, but are of doubtful validity when applied to skew-symmetrical loading problems.

the theory are reproduced in Appendix I where it is shown that the rolling moment coefficient is given by

$$-C_{\ell} = \beta \Gamma \frac{a_s}{2} \frac{4}{3\pi} (1 - y_d^2)^{3/2} , \qquad ....7$$

or

$$-\frac{\ell_v}{\Gamma} = \frac{a_s}{2} \frac{4}{3\pi} (1 - y_d^2)^{3/2} , \qquad ....8$$

where

$$a_s = \frac{1}{\frac{1}{a_\infty} + \frac{2}{\pi . A\!\!R}}$$

and $y_d s$ = distance along span where dihedral begins (see Fig. 1b).

For a full span dihedral we have $y_d = 0$, and

$$-\frac{\ell_{v_1}}{\Gamma} = \frac{a_s}{2} \frac{4}{3.\pi} ,$$

and, hence,

$$\frac{\ell_v/\Gamma}{\ell_{v_1}/\Gamma} = (1 - y_d^2)^{3/2} . \qquad ....9$$

The function $(1 - y_d^2)^{3/2}$ is shown in Fig. 2. From the form of equation 9 this function may be regarded as a measure of the rolling moment due to the dihedral extending from the point y_d to the wing tip. In Fig. 3 is shown the function denoted

$$-\eta_Y = -\frac{\partial}{\partial y} (1 - y^2)^{3/2} = 3y (1 - y^2)^{1/2} . \qquad ....10$$

This function is a measure of the contributions to the rolling moment coefficient of an element of dihedral at the point y, and hence may be regarded as a dihedral efficiency as far as rolling moment is concerned. It will be seen that this efficiency reaches a maximum of $y = 0.7$.

The only experimental results available for comparison with equation 9, are the results of some tests made by Shortal[15] on rectangular wings with well rounded tips. His results for ℓ_v/ℓ_{v_1} are shown plotted in Fig. 2. It will be seen that the agreement with the theoretical curve (equation 9) is remarkably close; this suggests that not only is the theoretical curve reliable but the ratio ℓ_v/ℓ_{v_1} is only slightly affected by taper. The effect of taper will be described in more detail later.

3.2. Effect of aspect ratio and two-dimensional lift curve slope (elliptic wing)

From the form of equation 8 we see that the aspect ratio and two dimensional lift curve slope (a_∞) occur only in the term a_s. If $\ell_v(\mathit{AR}, a_\infty)$ denote the value of ℓ_v for a given value of the aspect ratio AR and a given value of a_∞, then we can write

$$\frac{\ell_v(\mathit{AR}, a_\infty)}{\ell_v(6, a_\infty)} = \frac{1 + \frac{2\,a_\infty}{6.\pi}}{1 + \frac{2\,a_\infty}{\mathit{AR}.\pi}} = \frac{F(\mathit{AR}, a_\infty)}{F(6, a_\infty)}, \quad \text{say.} \qquad \ldots\ldots\ldots\ldots 11$$

We are here using as a convenient measure the rolling moment corresponding to an aspect ratio of 6.0. If we assume that, as an average value for most normal aerofoil sections, $a_\infty = 5.5$, then we have

$$\frac{F(\mathit{AR}, 5.5)}{F(6, 5.5)} = \frac{1.584}{1 + \frac{3.5}{\mathit{AR}}} \qquad \ldots\ldots\ldots\ldots 12$$

The ratio $F(\mathit{AR}, 5.5)/F(6, 5.5)$ given by equation 12 is shown in Fig.6a. For comparison the corresponding ratio for the lift curve slope of a wing in straight unyawed flight with symmetrical loading is shown dotted in the same figure.

We can allow for the fact that a_∞ varies with the type of aerofoil section used by computing the ratio $F(\mathit{AR}, a_\infty)/F(6, 5.5)$ for a range of values of a_∞ and AR. In other words, we can refer wings of all aspect ratios and values of a_∞ to a standard wing of aspect ratio 6 and for which $a_\infty = 5.5$. Thus we readily find that

$$\frac{F(\mathit{AR}, a_\infty)}{F(6, 5.5)} = \frac{\frac{1}{5.5} + \frac{1}{3\pi}}{\frac{1}{a_\infty} + \frac{2}{\mathit{AR}.\pi}} = \frac{0.288.a_\infty}{1 + \frac{0.637.a_\infty}{\mathit{AR}}} \quad . \qquad \ldots\ldots\ldots\ldots 13$$

The curves of this ratio corresponding to values of a_∞ of 5.0 and 6.0 (in addition to the standard value of 5.5) are shown in Fig.6b. For any given wing section the value of a_∞ can be determined either experimentally or by an empirical method such as is given in ref.24.

3.3. Tapered wings. Effect of dihedral span, aspect ratio and two dimensional lift curve slope

Hartshorn's calculations[5] on the rolling moment accompanying a part span skew symmetric incidence distribution include the case of the rectangular wing ($\tau = 1.0$), and we also have available the results of calculations by Pearson[6] on tapered wings for which τ varies from 0.25 to 1.0. The theory does not lead, as in the case of the elliptic wing to a formula for the rolling moment in which the effects of the span of the skew symmetric incidence distribution (i.e. dihedral span) and aspect ratio can be explicitly separated. A graphical analysis of the computed results of Pearson and Hartshorn demonstrates, however, that these effects are substantially separable. In Fig.5 the ratio

$\frac{\ell_v}{\Gamma} / \frac{\ell_{v_1}}{\Gamma}$ is plotted against dihedral span for the extreme taper ratios considered, the corresponding curve for the elliptic wing is shown dotted. It will be seen that there is little difference between the various curves and for most normal taper ratios, or where very great accuracy is not desired, the curve for the elliptic wing can be reliably used. In Fig.7 the calculated values of the ratio $F(R, 5.5)/F(6, 5.5)$ are plotted against aspect ratio for values of the dihedral span corresponding to $y_d = 0, 0.4$ and 0.6, and the corresponding curve for the elliptical wing is also shown. It will be seen that the values of this ratio for the tapered wings depart very little from the curve for the elliptic wing except possibly for the rectangular wings, at large aspect ratios (12 or higher), and with small dihedral span, i.e. $y_d > 0.6$. For all practical purposes therefore it may be taken that the ratio $\frac{F(R, 5.5)}{F(6, 5.5)}$ is the same for tapered wings as for an elliptic wing. In other words, it appears from this analysis that the effect of dihedral span and aspect ratio are approximately the same for all wings of normal plan form whether elliptical or tapered. It seems reasonable to suppose that the same conclusion applies as far as the effect of a_∞ is concerned.

4. Effect of spanwise variation in dihedral

4.1. General

We have seen how, for an elliptic wing with constant full or part span dihedral Γ, each spanwise element Δy contributes in sideslip the element of the rolling moment derivative

$$\Delta \ell_v = \frac{\Delta C_\ell}{\beta} = -\eta_\gamma \, \Gamma \, \frac{a_s}{2} \frac{4}{3\pi} \Delta y$$

where (see Fig.3)

$$\eta_\gamma = 3\, y(1 - y^2)^{1/2} .$$

We have called η_γ the dihedral efficiency.

Since the lifting line theory on which these formulae are based is a linear one, i.e. it assumes a linear relation between the local lift coefficient and the local incidence at any point along the span, the total rolling moment of a wing may be obtained as the sum total of the rolling moments due to the spanwise elements of the wing. In other words, the contribution of an element to the rolling moment due to the changes produced by this element in the spanwise loading distribution, is independent of the contributions produced by the remainder of the wing. This is true in general whatever the spanwise dihedral distribution of an elliptic wing. Hence, the rolling moment derivative in sideslip of an elliptic wing with spanwise dihedral distribution $\gamma(y)$ is given by

$$-\ell_v = -\frac{\partial C_\ell}{\partial \beta} = \frac{a_s}{2} \frac{4}{3\pi} \int_0^1 \eta_\gamma \; \gamma(y) \; dy$$

$$= \frac{a_s}{2} \frac{4}{3\pi} \int_0^1 \gamma(y) \, 3y(1 - y^2)^{1/2} \; dy \quad \ldots\ldots\ldots\ldots 14$$

4.2. Wing divided into various spanwise parts having different constant dihedral angles

4.2.1. Elliptic wing

We will consider a wing divided into three parts; the method can readily be extended to a wing divided into any number of parts. Let Υ_1, Υ_2 and Υ_3 be the dihedral angles*, and y_{d_1}, y_{d_2} and y_{d_3} the spanwise ordinates at the end of each part (see Fig.1d). Then

$$-\ell_v = -\frac{\partial C_\ell}{\partial \beta} = \frac{a_s}{2}\,\frac{4}{3\pi}\int_0^1 \Upsilon(y)\; 3y(1-y^2)^{1/2}\,dy$$

$$= \frac{a_s}{2}\,\frac{4}{3\pi}\left\{\int_0^{y_{d_1}} \Upsilon_1\; 3y(1-y^2)^{1/2}\,dy + \int_{y_{d_1}}^{y_{d_2}} \Upsilon_2\; 3y(1-y^2)^{1/2}\,dy\right.$$

$$\left. + \int_{y_{d_2}}^{y_{d_3}} \Upsilon_3\; 3y(1-y^2)^{1/2}\,dy\right\}$$

$$= \frac{a_s}{2}\,\frac{4}{3\pi}\left\{\Upsilon_1\left[1-(1-y_{d_1}{}^2)^{3/2}\right] + \Upsilon_2\left[(1-y_{d_1}{}^2)^{3/2} - (1-y_{d_2}{}^2)^{3/2}\right] +\right.$$

$$\left. + \Upsilon_3\left[(1-y_{d_2}{}^2)^{3/2} - (1-y_{d_3}{}^2)^{3/2}\right]\right\}$$

$$= \frac{a_s}{2}\,\frac{4}{3\pi}\left\{\Upsilon_1 + (1-y_{d_1}{}^2)^{3/2}(\Upsilon_2-\Upsilon_1) + (1-y_{d_2}{}^2)^{3/2}(\Upsilon_3-\Upsilon_2)\right\}, \qquad \ldots\ldots.15$$

since in the case considered $y_{d_3} = 1.0$. The functions $(1 - y_{d_1}{}^2)^{3/2}$ and $(1 - y_{d_2}{}^2)^{3/2}$ can be read off the curve of Fig.2. We can write equation 15 in the form

$$\ell_v = \left(\frac{\ell_{v_1}}{\Gamma}\right)_{ell.} \frac{F(R, a_\infty)}{F(6, 5.5)} \left\{\Upsilon_1 + (1-y_{d_1}{}^2)^{3/2}(\Upsilon_2 - \Upsilon_1) +\right.$$

$$\left. + (1-y_{d_2}{}^2)^{3/2}(\Upsilon_3 - \Upsilon_2)\right\}, \qquad \ldots\ldots\ldots\ldots.16$$

---

* Any of Υ_1, Υ_2, Υ_3 etc. can be negative, i.e. the corresponding part of the wing has anhedral.

where $\left(\frac{\ell_{v_1}}{\Gamma}\right)_{ell.}$ refers to aspect ratio 6 and $a_\infty = 5.5$ and is therefore equal to 0.74. Equation 16 could be derived directly from equation 7.

4.2.2. Tapered wings

For tapered wings we similarly obtain

$$\ell_v = \left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau \frac{F(R\,,\,a_\infty)}{F(6,\,5.5)} \Big\{ \gamma_1 + f(y_{d_1},\,\tau)\,(\gamma_2 - \gamma_1) + $$

$$+ f(y_{d_2},\,\tau)\,(\gamma_3 - \gamma_2) \Big\} \quad , \qquad \ldots\ldots\ldots\ldots 17$$

where $f(y_d,\,\tau)$ is obtained from Fig.5,

$\left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau$ is given in Fig.4 (curve B),

and $\frac{F(R\,,\,a_\infty)}{F(6,\,5.5)}$ can be taken from the curves given in Fig.6.

It will be appreciated that the above formulae (16 and 17) include the case where the wing tip has a dihedral angle different from that of the adjacent part of the wing (see Fig.1d).

4.3. Dihedral effect of wing flexure under the aerodynamic loads in flight.

4.3.1. Elliptic wing

Let δ_z be the local displacement parallel to the Z axis (reversed) of the local quarter chord point due to the wing flexure under the aerodynamic loads in flight. In practice, it is found that the relation between δ_z and y is adequately expressed by

$$\delta_z = k.y^n \,,$$

where n varies between 1.6 and 2.0 (ref.26).

If δ_{z_1} is the displacement at the tip, then

$$\delta_z/\delta_{z_1} = y^n . \qquad \ldots\ldots\ldots\ldots 18$$

The corresponding dihedral angle $\gamma(y)$ at the point y is given by

$$\gamma(y) = \frac{1}{s}\,\frac{\partial \delta_z}{\partial y} = \left(\frac{\delta_{z_1}}{s}\right) \; n\,y^{n-1} . \qquad \ldots\ldots\ldots\ldots 19$$

Hence from equation 14 the rolling moment derivative in sideslip due to the wing flexure dihedral is

$$-\ell_v = -\frac{\partial C_\ell}{\partial \beta} = \frac{\delta_{s_1}}{s} \frac{a_s}{2} \frac{4n}{3\pi} \int_0^1 3y^n (1 - y^2)^{1/2} dy .$$

If we write $\frac{\delta_{s_1}}{s} = \Gamma_\delta$ (i.e. the constant full span dihedral angle associated with the same tip deflection) then

$$-\frac{\ell_v}{\Gamma_\delta} = \frac{a_s}{2} \frac{4n}{3\pi} \int_0^1 3y^n (1 - y^2)^{1/2} dy.$$

We have already seen that for constant full span dihedral

$$-\frac{\ell_{v_1}}{\Gamma} = \frac{a_s}{2} \cdot \frac{4}{3\pi} ,$$

hence we may write

$$\left(\frac{\ell_v}{\Gamma} \Big/ \frac{\ell_{v_1}}{\Gamma}\right)_{ell.} = n \int_0^1 3y^n (1 - y^2)^{1/2} dy. \qquad \text{.........20}$$

The values of the integral on the right hand side of equation 20 have been calculated for a range of values of n from 0 to 6.0 and the results are shown in Fig. 8. It will be seen that the variation of the value of this integral is small for n greater than 1.6, and for the range of values of n found in practice (1.6 to 2.0), the integral may be taken to be 1.18.

Hence

$$\frac{\ell_v}{\Gamma} \simeq \frac{\ell_{v_1}}{\Gamma} \; 1.18. \qquad \text{............21}$$

For a wing of aspect ratio 6 and for which $a_\infty = 5.5$, we have seen that

$$\frac{\ell_{v_1}}{\Gamma} = -0.74,$$

and hence for such a wing when flexed

$$\frac{\ell_v}{\Gamma_\delta} = -0.88 ,$$

or

$$-(\ell_v)_{ell.} = 0.88 \frac{\delta_{s_1}}{s}. \qquad \text{............22}$$

For aspect ratios other than 6.0, the correction factor, as given in Fig.6, should be applied. We have thus arrived at a formula giving the derivative directly in terms of the tip deflection.

4.3.2. Tapered wings

The above discussion applies strictly to wings of elliptical plan form, but we may expect, from the very small effect that taper was found to have in the case of constant dihedral, that equation 21 will be sufficiently valid for wings of normal taper ratios. As a check, calculations have been made of the ratio $\ell_v/\Gamma_\delta \quad \ell_{v1}/\Gamma$

for wings of taper ranging from 0.25 to 1.0 and of aspect ratios 4, 6 and 8, the flexure being assumed to be parabolic (i.e. n = 2). With a parabolic flexure the dihedral angle, and hence the change in effective incidence ($\Delta\alpha$) varies linearly with spanwise distance from the rolling axis. The theory is therefore identical with the theory for a wing rolling with constant angular velocity in which the helical character of the trailing vortices is ignored. Calculations for this latter case covering the taper ratios and aspect ratios quoted above are given in ref.6. It is a simple matter therefore to apply the results of these calculations to give us the ratio $\frac{\ell_v}{\Gamma_\delta}/\frac{\ell_{v1}}{\Gamma}$. The details are described in Appendix II. The variation of this ratio (denoted $f(\tau)_\delta$) is shown in Fig.9. It will be seen that for values of $\tau > 0.3$ and for the range of aspect ratios considered the effect of taper and aspect ratio is small, and for most practical purposes equation 21, though derived for wings of elliptical plan form, may be used for tapered wings.

5. Comparison with experiment

A number of experimental results derived from wind tunnel tests on a variety of model wings and complete model aeroplanes, including two swept back wings, have been examined in the light of the theoretical results described above. In each case the measured value of ℓ_v/Γ was converted to the standard aspect ratio 6.0 and the standard value of $a_\infty = 5.5$ by means of the curves of Fig.6. The result was then converted to a full span dihedral by means of the curves of Fig.2. The final value of ℓ_{v1}/Γ is plotted against τ in Fig.4. It will be seen that the scatter of the resulting points is gratifyingly small and a mean line (marked E) can be readily drawn through these points. This line can be represented by the empirical equation

$$\left(\frac{\ell_{v1}}{\Gamma}\right)_\tau = 0.65 + 0.17\,\tau \qquad \text{............23}$$

The curve marked A (in Fig.4) has been determined wholly from the theoretical calculations of Pearson[6] and Hartshorn[5]. The small scatter of the experimental points and the close agreement between curves A and E provide a valuable confirmation of the theory.

Some remarks on the surprisingly small effect of taper ratios on rolling moment as indicated by the small slope of the line E, would not be out of place here. If we revert to the crude theoretical approach, in which induced effects due to the skew symmetric incidence changes are neglected, and the change of lift with incidence is assumed to be the same as for symmetrical loading, then it is easy to see that the rolling moment should be proportional to the first moment of the area of each half wing.

We then find that

$$\frac{(\ell_v)_\tau}{(\ell_v)_{\tau=1}} = 1 - \frac{2}{3}(1-\tau). \qquad \text{..............24}$$

The curve marked B in Fig.4 has been derived on the basis of this relation, and the marked disparity between the slopes of curve B and those of A or E will be apparent. It follows therefore, that the large effect of taper ratio, that one tends to expect on the basis of arguments similar to that of the crude theory, is almost entirely nullified by the accompanying induced changes of downwash, which are therefore much too important to be neglected.

6. Conclusions, summary of main formulae and example

6.1. Conclusions

6.1.1. The effect of dihedral in sideslip is equivalent to that of a skew-symmetric change in the incidence distribution in unyawed flight, such as is caused by a non-differential movement of the ailerons. The theory of the rolling moment caused by aileron movement is well developed, and readily transformed to apply to the case of dihedral in sideslip.

6.1.2. For wings of elliptic plan form the rolling moment is proportional to the dihedral and sideslip angles (assumed small), to a function of the dihedral span (Fig.2) and to a function of the aspect ratio (Fig.6a). The form of this latter function is the same as that controlling the change (with aspect ratio) of the lift curve slope in straight unyawed flight, except that it relates to an effective aspect ratio half of the geometric aspect ratio.

6.1.3. For tapered wings the same conclusions apply, and in fact for rough estimates the effect of taper may be ignored and wings of normal plan form may be treated as elliptical. More accurately, the effect of dihedral span and the variation of the constant of proportionality, i.e. ℓ_{v1}/Γ, with taper can be obtained from Figs.5 and 4.

6.1.4. Comparison of the theoretical deductions with the available experimental results shows very satisfactory agreement (Fig.4); this indicates that sweepback and interference due to the presence of a fuselage or nacelles have negligible effects on the contribution to ℓ_v due to wing dihedral.

6.1.5. The contribution to ℓ_v caused by wing flexure under the aerodynamic loads in flight is simply expressible in terms of tip deflection, and in practice, this effect is only slightly dependent on wing taper and can be readily determined for the type of flexure that normally occurs (nearly parabolic); (Figs.8 and 9).

6.2. Summary of main formulae

The main formulae for ℓ_v due to wing dihedral derived in this note are here summarised as follows:

6.2.1. Uniform full and part span dihedral

(i) For elliptic wing

$$-\frac{\ell_v}{\Gamma} = 0.74\ (1 - y_d{}^2)^{3/2}\ \frac{F(A, a_\infty)}{F(6, 5.5)}$$

(ii) For tapered wings

$$-\frac{\ell_v}{\Gamma} = -\left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau\ f(y_d, \tau)\ \frac{F(A, a_\infty)}{F(6, 5.5)}$$

where,

$$-\left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau = 0.65 + 0.17\,\tau \text{ (see Fig.4)},$$

$f(y_d, \tau)$ is found from Fig.5, and

$\frac{F(A, a_\infty)}{F(6, 5.5)}$ is a correction factor for aspect ratio and two dimensional lift curve slope and is given in Fig.6.

6.2.2. Variable dihedral distribution $[\Upsilon(y)]$

(i) Elliptic wing (general)

$$-\ell_v = 0.74\ \frac{F(A, a_\infty)}{F(6, 5.5)} \int_0^1 \Upsilon(y).3y\ (1 - y^2)^{1/2}\ dy.$$

(ii) Elliptic wing (three spanwise parts of constant dihedral differing from each other)

$$-\ell_v = 0.74\ \frac{F(A, a_\infty)}{F(6, 5.5)} \left\{ \Upsilon_1 + (1 - y_{d_1}{}^2)^{3/2}\ (\Upsilon_2 - \Upsilon_1) + (1 - y_{d_2}{}^2)^{3/2}\ (\Upsilon_3 - \Upsilon_2) \right\}.$$

(iii) Tapered wings (three spanwise parts of constant dihedral differing from each other).

$$\ell_v = -\left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau \frac{F(A, a_\infty)}{F(6, 5.5)} \left\{ \gamma_1 + f(y_{d_1}, \tau)\,(\gamma_2 - \gamma_1) + \right.$$

$$\left. + f(y_{d_2}, \tau)\,(\gamma_3 - \gamma_2) \right\} .$$

(iv) Elliptic wing (flexed, where $1.6 \leq n \leq 5$)

$$-\ell_v = 0.88 \frac{F(A, a_\infty)}{F(6, 5.5)} \frac{\delta_{z_1}}{s} .$$

(v) Tapered wings (flexed)

Formula 5.2.2.(iv) above is in general accurate enough, but more precisely, when $n \approx 2$, we have

$$-\ell_v = -\left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau f(\tau)_\delta \frac{F(A, a_\infty)}{F(6, 5.5)} \frac{\delta_{z_1}}{s} .$$

where $f(\tau)_\delta$ is given in Fig.9.

6.3. Example

Suppose we require the rolling moment derivative ℓ_v due to the total dihedral effect in straight (yawed) flight for the wing shape shown in Fig.1d and with the following characteristics.

$b_1 = 22$ ft.
$b_2 = 54$ ft.
$b = 60$ ft.
$\tau = 0.3$
$A = 8.0$
$a_\infty = 5.7$

$\gamma_1 = -5^\circ = -0.087$ rad.
$\gamma_2 = 3^\circ = 0.052$ rad.
$\gamma_3 = 8^\circ = 0.140$ rad.

For straight flight, i.e. $\frac{\text{aerodynamic load}}{\text{total weight}} = 1$,

$$\frac{\delta_{z_1}}{s} = 0.015.$$

Then $y_{d_1} = 0.367$ and $y_{d_2} = 0.90$.

Formulae 5.2.2.(iii) and (v) (for tapered wings) give the total effect on ℓ_v due to dihedral angles Υ_1, Υ_2 and Υ_3 and wing flexure.

$$\text{From Fig. 4 (curve E)} \quad \left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau = -0.701,$$

$$\text{from Fig. 6} \quad \frac{F(A\!R,\ a_\infty)}{F(6,\ 5.5)} = \frac{F(8,\ 5.7)}{F(6,\ 5.5)} = 1.105,$$

$$\text{from Fig. 5} \quad f(y_{d_1},\ \tau) = 0.783,\ f(y_{d_2},\ \tau) = 0.097$$

$$\text{and from Fig. 9} \quad f(\tau)_\delta = 1.15.$$

Hence, using the above formulae for tapered wings we have

$$-\ell_v = -\left(\frac{\ell_{v_1}}{\Gamma}\right)_\tau \frac{F(A\!R,\ a_\infty)}{F(6,\ 5.5)} .$$

$$.\left\{\left[\Upsilon_1 + f(y_{d_1},\tau)(\Upsilon_2-\Upsilon_1) + f(y_{d_2},\ \tau)\ (\Upsilon_3-\Upsilon_2)\right] + f(\tau)_\delta\ \frac{\delta_{s_1}}{s}\right\}$$

$$= 0.701\ .\ 1.105\left\{\left[-0.087 + 0.783\ (0.052 + 0.087) +\right.\right.$$

$$\left.\left. + 0.097\ (0.14 - 0.052)\right] + 1.15\ .\ 0.015\right\}$$

$$= \underline{0.037}$$

If expressions 5.2.2.(ii) and (iv) corresponding to an elliptic wing are used instead of those for tapered plan form, the following result is obtained

$$-\ell_v = 0.74\ \frac{F(A\!R,\ a_\infty)}{F(6,\ 5.5)}\left\{\Upsilon_1 + (1-y_{d_1}{}^2)^{3/2}\ (\Upsilon_2-\Upsilon_1) + (1 - y_{d_2}{}^2)^{3/2}\ (\Upsilon_3-\Upsilon_2)\right.$$

$$\left. + 1.18\ \frac{\delta_{s_1}}{s}\right\} = 0.041.$$

It will be appreciated that the difference of about 10% between the above estimates of ℓ_v would not be present if a wing with a taper ratio of about 0.5 had been considered instead of 0.3

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Attached:

Appendices I and II

Fig. 1	Drg.	No.	16444S
" 2 and 3	"	"	16445S
" 4 and 5	"	"	16446S
" 6a	"	"	16447S
" 6b	"	"	16448S
" 7	"	"	16449S
" 8 and 9	"	"	16450S

Circulation:

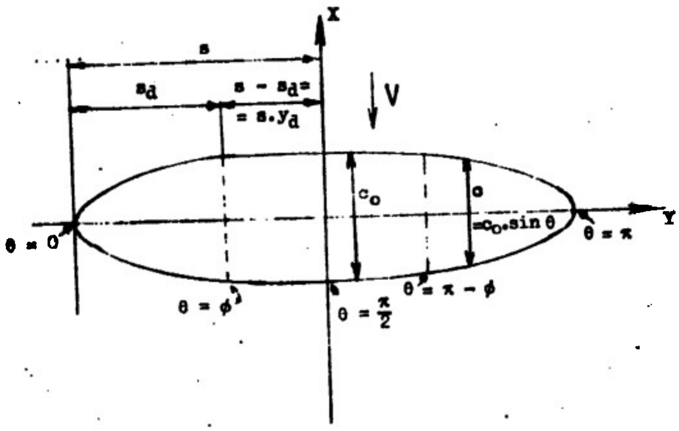
C.R.D.	
D.S.R.	
D.D.S.R.1.	(Action copy)
D.D./R.D.T.	
A.D./R.D.T.1.	
S.R.1.	
A.D.R.D.L.1.	(2)
A.D.R.D.L.2.	(2)
R.T.P.(T.I.B.)	(6+1)
R.T.P.	(110)
A.& A.E.E.	(2)
A.R.C.	(40)

Appendix I

1. Rolling moment due to a skew symmetric incidence distribution of constant magnitude (full and part span)

1.1. Elliptic wing

No originality can be claimed for the following theory, Munk[2], Scarborough[4] and Hartshorn[5] are in the main responsible for its development. Its salient features are reproduced here for the sake of completeness. It is believed, however, that the application of the theory to the determination of the rolling moment due to dihedral in sideslip is somewhat novel.



Consider an elliptic wing of span 2s and maximum chord c_o. We define the parameter θ by the relation

$$y_s = - s \cos \theta, \qquad \text{......25a}$$

$$c = c_o \sin \theta, \text{ for an elliptic wing,} \qquad \text{......25b}$$

where c is the local chord corresponding to y the spanwise ordinate (expressed as a fraction of the semi-span).

We assume the circulation $K(\theta)$ is given by the Fourier series

$$K(\theta) = 4s\,V\,\Sigma\,A_n \sin n\,\theta. \qquad \text{......26}$$

Then it is readily shown, as in standard text books, (e.g. ref. 12), that the downwash at the wing w is given by

$$w \sin\theta = V \Sigma n A_n \cdot \sin n\theta. \qquad \ldots\ldots\ldots\ldots\ldots 27$$

The basic relation connecting the local lift coefficient C_L and circulation at any spanwise station is

$$K(\theta) = C_L \frac{c}{2} V. \qquad \ldots\ldots\ldots\ldots\ldots 28$$

But C_L is related to the local incidence $(\alpha - {}^w/_V)$ by

$$C_L = a_\infty \left(\alpha - \frac{w}{V}\right), \qquad \ldots\ldots\ldots\ldots\ldots 29$$

where α is the effective geometric incidence.

If we suppose α to be composed of a part span skew symmetric distribution of constant magnitude superposed on a constant incidence α_0, we can write

$$\left.\begin{aligned} &\alpha = \alpha_0 + f(\theta)\,\Delta\alpha \,, \\ &\text{where } f(\theta) = 1.0 \text{ from } \theta = 0 \text{ to } \theta = \phi, \text{ say,} \\ &\qquad = 0 \text{ from } \theta = \phi \text{ to } \theta = \pi - \phi \,, \\ &\qquad = -1.0 \text{ from } \theta = \pi - \phi \text{ to } \theta = \pi \,. \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots 30$$

From 28 and 29 we have

$$K(\theta) = \frac{cV}{2}\, a_\infty \left(\alpha - \frac{w}{V}\right),$$

and substituting for w and $K(\theta)$ from equations 26 and 27, we find

$$\Sigma A_n \sin n\theta\,(n\mu_0 + 1) = \mu_0\,\alpha \sin\theta,$$

$$\text{where } \mu_0 = \frac{a_\infty c_0}{8s} \,. \qquad \ldots\ldots\ldots\ldots\ldots 31$$

Hence

$$\int_0^\pi \Sigma A_n \sin n\theta \,(n\mu_o + 1) \sin n\theta \, d\theta = \int_0^\pi \mu_o \alpha \;\sin\theta \sin n\theta \, d\theta$$

$$= \int_0^\pi \mu_o \alpha_o \sin\theta \sin n\theta \, d\theta \, +$$

$$+ \int_0^\pi \mu_o f(\theta) \,\Delta\alpha \;\sin\theta \sin n\theta \, d\theta.$$

Therefore: for n = 1

$$A_n (n \mu_o + 1) \frac{\pi}{2} = \mu_o \alpha_o \frac{\pi}{2} ,$$

and for $n \neq 1$

$$A_n (n\mu_o + 1) \frac{\pi}{2} = \frac{\mu_o \Delta\alpha}{2} \left[\frac{\sin (n-1)\phi}{n-1} - \frac{\sin (n+1)\phi}{n+1} + \right.$$

$$\left. + \frac{\sin \overline{(n-1)(\pi-\phi)}}{n-1} - \frac{\sin \overline{(n+1)(\pi-\phi)}}{n+1} \right] .$$

Hence* $A_1 = \frac{\mu_o}{(\mu_o + 1)} \cdot \alpha_o$, $A_3 = A_5 = \ldots\ldots = 0,$

$$A_{2n} = \frac{2}{\pi} \; \frac{\mu_o \Delta\alpha}{(2\,n\,\mu_o + 1)} \left[\frac{\sin (2n-1)\phi}{2n-1} - \frac{\sin (2n+1)\phi}{2n+1} \right] . \qquad \ldots\ldots\ldots\ldots 32$$

The local c_L at any point

$$= \frac{2\,K(\theta)}{cV} = \frac{8s}{c} \;\Sigma A_n \sin n\,\theta ,$$

*In general, for any combination of symmetric and skew symmetric incidence distributions, the odd terms derive from the symmetric and the even terms from the skew symmetric distributions.

hence the rolling moment coefficient is

$$-\frac{s^2}{S}\int_0^1 c_L \frac{c}{s}\, y\, dy = -\frac{1}{Ss}\int_0^{\frac{\pi}{2}} \left(\frac{8s}{c}\, \Sigma A_n \sin n\theta\right) y\, s^2 c \frac{dy}{d\theta}\, d\theta$$

$$= \frac{4s^2}{S}\int_0^{\frac{\pi}{2}} (\Sigma A_n \sin n\theta) \sin 2\theta\, d\theta$$

$$= \frac{\pi}{4} A_2 \mathcal{A} \qquad \text{.............33}$$

$$= \frac{\mathcal{A}\, \mu_0\, \Delta\alpha}{2(2\mu_0 + 1)} \left[\sin\phi - \frac{\sin 3\phi}{3}\right] . \qquad \text{.............34}$$

But we can write

$$\mu_0 = \frac{a_\infty}{\pi \mathcal{A}} ,$$

hence

$$\frac{\mathcal{A}\, \mu_0}{2(2\mu_0 + 1)} = \frac{1}{2\pi}\left[\frac{1}{\frac{1}{a_\infty} + \frac{2}{\pi . \mathcal{A}}}\right]$$

$$= \frac{a_s}{2\pi} , \quad \text{say.} \qquad \text{.............35}$$

Also $\left(\sin\phi - \frac{\sin 3\phi}{3}\right) = \frac{4}{3} \sin^3\phi$

$$= \frac{4}{3}\left(1 - y_d^2\right)^{3/2} .$$

Hence, the rolling moment coefficient is

$$C_\ell = \frac{a_s}{2}\, \frac{4\,\Delta\alpha}{3\pi}\left(1 - y_d^2\right)^{3/2} . \qquad \text{.............36}$$

For a wing with dihedral Γ in sideslip, the sideslip angle being β, the change in incidence is given by

$$\left|\tan\ (\Delta\alpha)\right| = \tan\beta \sin\Gamma, \qquad \text{..........37}$$

or $\Delta\alpha \approx \beta\ \Gamma$

if β and Γ are small and measured in radians.

Hence, from 36

$$-\frac{\ell_v}{\Gamma} = -\frac{\partial C_\ell}{\partial\beta}\Big/_{\Gamma} = \frac{a_s}{2}\ \frac{4}{3\pi}\ (1 - y_d^{\,2})^{3/2}. \qquad \text{.......38}$$

For a full span dihedral $y_d = 0$, and hence

$$-\frac{\ell_{v_1}}{\Gamma} = \frac{a_s}{2}\ \frac{4}{3\pi}. \qquad \text{..........39}$$

It follows that

$$\frac{\ell_v}{\Gamma}\Big/\frac{\ell_{v_1}}{\Gamma} = (1 - y_d^{\,2})^{3/2} \qquad \text{..........40}$$

The function $(1 - y_d^{\,2})^{3/2}$ therefore is the ratio of the rolling moment in sideslip due to part span dihedral to that due to full span dihedral. It is plotted in Fig. 2. The quantity a_s determines the aspect ratio effect, and is shown in Fig. 6.

1.2. Tapered wings

The above theory for elliptic wings is readily extended to tapered wings (see for example refs. 5, 6 and 7) and the details need not be reproduced here. The rolling moment coefficient can again be derived in the form of equation 33 above,

$$-C_\ell = \frac{\pi}{4}\ \frac{A_2}{\Delta\alpha}\ R\ \ \Delta\alpha, \qquad \text{..........41}$$

where $\frac{A_2}{\Delta\alpha}$ cannot now be expressed in a simple analytic form, as was possible above, for the elliptic wing, but must be determined numerically. Using the calculations of refs. 5, 6 and 7 the curves of Fig. 5 and Fig. 7 have been derived for the effects of dihedral span and aspect ratio. In each case it will be seen that for normal taper ratios the curves are very close to those derived above for a wing of elliptical plan form.

Appendix II

1. Relationship between a wing with parabolic flexure (n = 2) and a wing of the same plan form with constant dihedral and having the same tip deflection (tapered wings)

For a wing with parabolic flexure, the deflection δ_z at the spanwise position y is given by

$$\frac{\delta_z}{\delta_{z_1}} = y^2 . \qquad \text{.............42}$$

Hence, the local dihedral angle $\Upsilon(y)$ is given by

$$\Upsilon(y) = \frac{1}{s} \frac{\partial \delta_z}{\partial y} = 2 \frac{\delta_{z_1}}{s} y = 2 \Gamma_\delta y, \qquad \text{.............43}$$

where $\Gamma_\delta = {}^{\delta_{z_1}}/_{s}$ 44

In sideslip the corresponding change in effective incidence is

$$\Delta\alpha (y) = \Upsilon(y) \beta = 2 \Gamma_\delta y \beta . \qquad \text{.............45}$$

Consider the same wing without flexure but with a linear skew symmetric change in distribution of incidence due to, say, rolling at constant angular velocity p. Then the change in the effective incidence at the spanwise station y due to the roll is

$$\Delta\alpha_p (y) = \frac{p\, sy}{V} \qquad \text{.............46}$$

Hence, the distributions of the changes in effective incidence in the two cases are identical if we have

$$\Delta\alpha_{p1} = \frac{ps}{V} = 2 \Gamma_\delta \beta , \qquad \text{.............47}$$

where $\Delta\alpha_{p1}$ is the change in the effective local incidence at the tip due to rolling.

Pearson has developed[6] the analysis of tapered wings rolling with uniform angular velocity p, on the assumption that the distortion of the trailing vortices could be neglected, reducing the problem therefore to that of the wing in straight unyawed flight with a linear skew symmetric change in the incidence distribution. In ref. 6 it is shown that the rolling moment coefficient satisfies the equation

$$C_{\ell} = \frac{\pi}{4} A \left[\frac{a_2'}{\Delta\alpha_{p1}} \right] \Delta\alpha_{p1} , \qquad \text{.............48}$$

where A_2' is the coefficient of $\sin 2\theta$ in the Fourier series assumed in the analysis to describe the circulation distribution.

We have seen that similarly for a wing with a constant dihedral angle Γ_δ, the rolling moment coefficient in sideslip is given by

$$C_\ell = \frac{\pi}{4} A \left[\frac{A_2}{\Gamma_\delta \beta}\right] \Gamma_\delta \; \beta \; , \qquad \text{............49}$$

where A_2 is, likewise, the coefficient of $\sin 2\theta$ in the corresponding circulation Fourier series. If we write

$$\frac{A_2}{\Gamma_\delta \beta} = F_2, \quad \text{and} \quad \frac{A_2'}{\Delta\alpha_{P_1}} = F_2' \; ,$$

then the ratio of the rolling moments in the two cases is

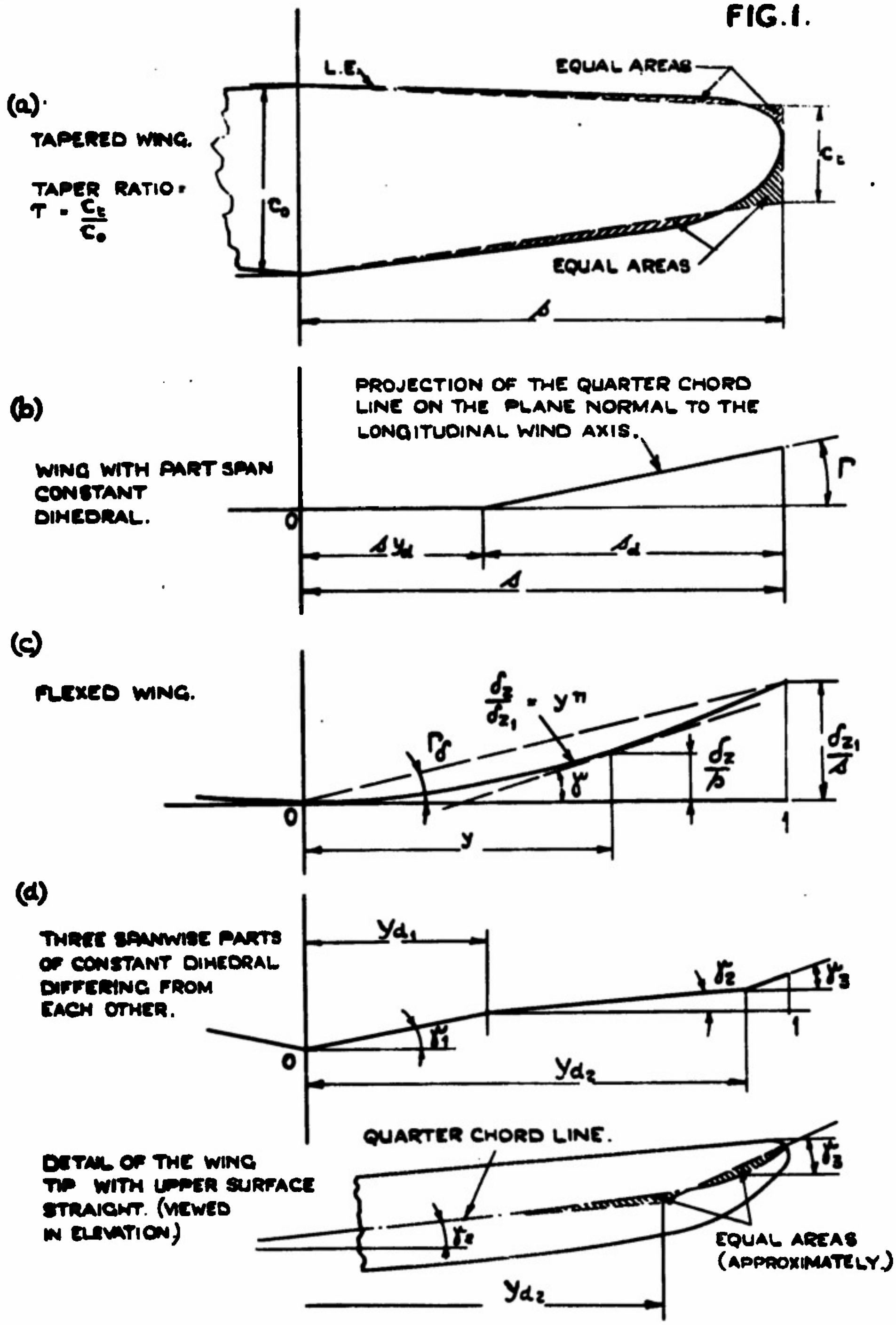
$$\frac{F_2'}{F_2} \; \frac{\Delta\alpha_{P_1}}{\Gamma_\delta \beta} \; .$$

If we now replace the rolling wing by the wing with the equivalent parabolic flexure, i.e. replace $\Delta\alpha_{P_1}$ by $2\,\Gamma_\delta\,\beta$ (equation 45), then the ratio of the rolling moment becomes

$$\left(\frac{\ell_v}{\Gamma_\delta} \Big/ \frac{\ell_{v_1}}{\Gamma}\right)_\tau = \frac{2\,F_2'}{F_2} = f(\tau)_\delta \; , \quad \text{say.} \qquad \text{............50}$$

This ratio is therefore the ratio of the rolling moment coefficient for unit sideslip of a parabolically flexed wing to the rolling moment coefficient for unit sideslip of the unflexed wing with full span constant dihedral giving the same tip displacement. From the calculations of ref.6 the ratio $2\,F_2'/F_2$ was determined for taper ratios of 0.25, 0.50, 0.75 and 1.0 and for aspect ratios of 4, 6 and 8 and the results are shown in Fig.9.

FIG. 1.



NOTATION FOR WING WITH DIHEDRAL AND FLEXTURE.

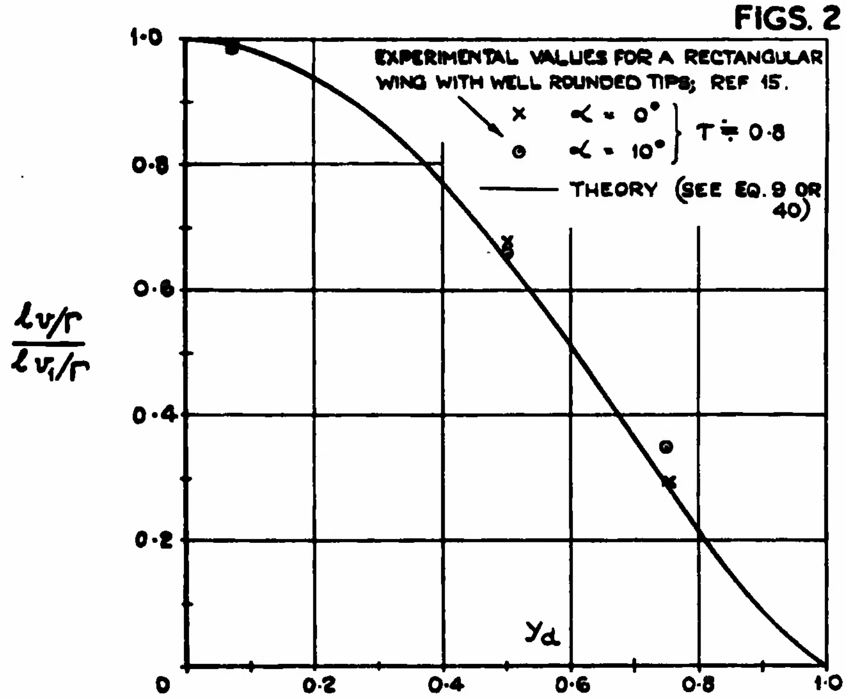


FIG. 2.

EFFECT OF A PART SPAN CONSTANT DIHEDRAL DISTRIBUTION ON $\frac{lv/\Gamma}{lv_1/\Gamma}$ FOR AN ELLIPTIC WING.

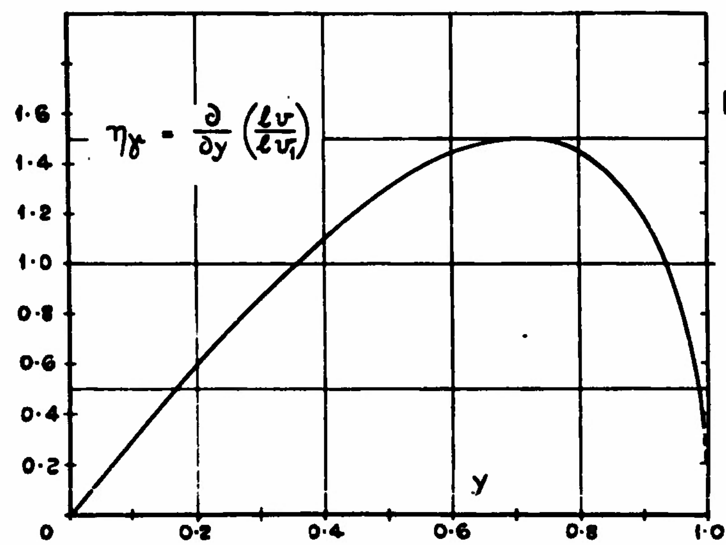
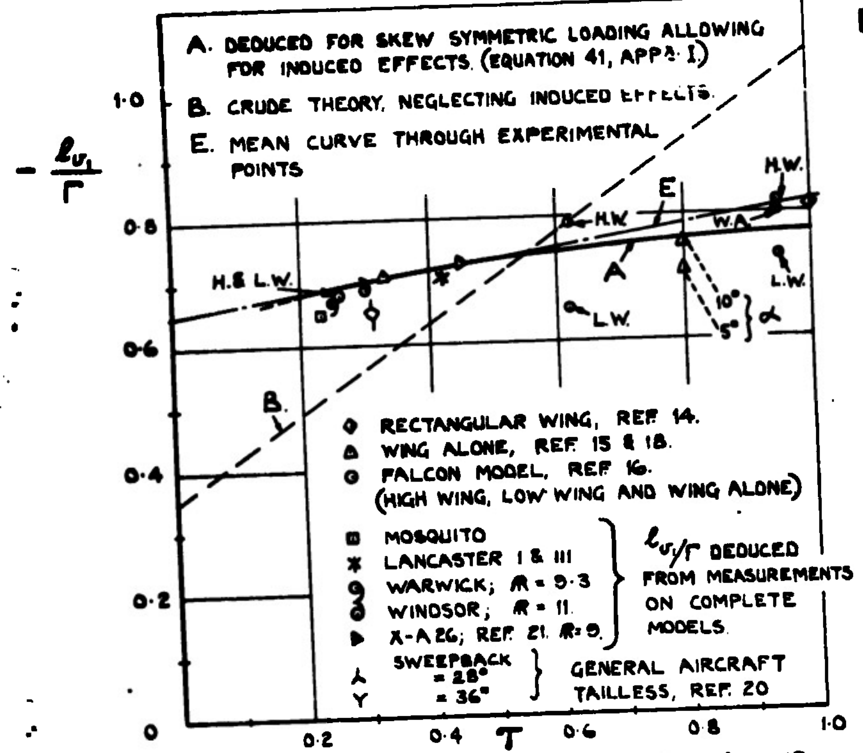


FIG. 3.

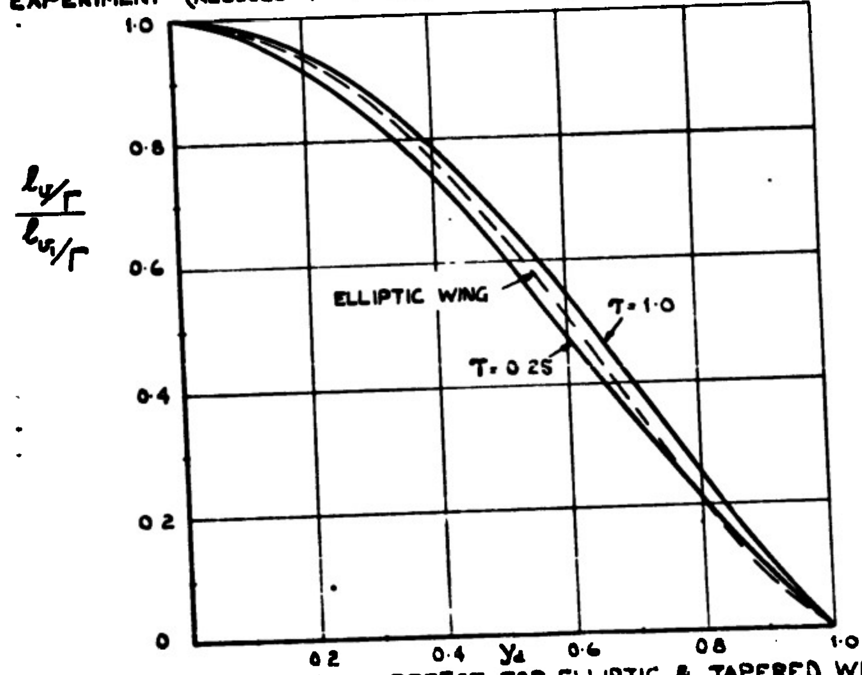
VARIATION OF THE DIHEDRAL EFFICIENCY η_γ ALONG THE SEMI-SPAN FOR AN ELLIPTIC WING.

FIG. 4



TAPER EFFECT COMPARISON BETWEEN THEORY AND EXPERIMENT (REDUCED TO STANDARD CONDITION, i.e. A = 6 AND a_∞ = 5·5; y_d = 0)

FIG. 5



DIHEDRAL SPAN EFFECT FOR ELLIPTIC & TAPERED WINGS. (THEORETICAL)

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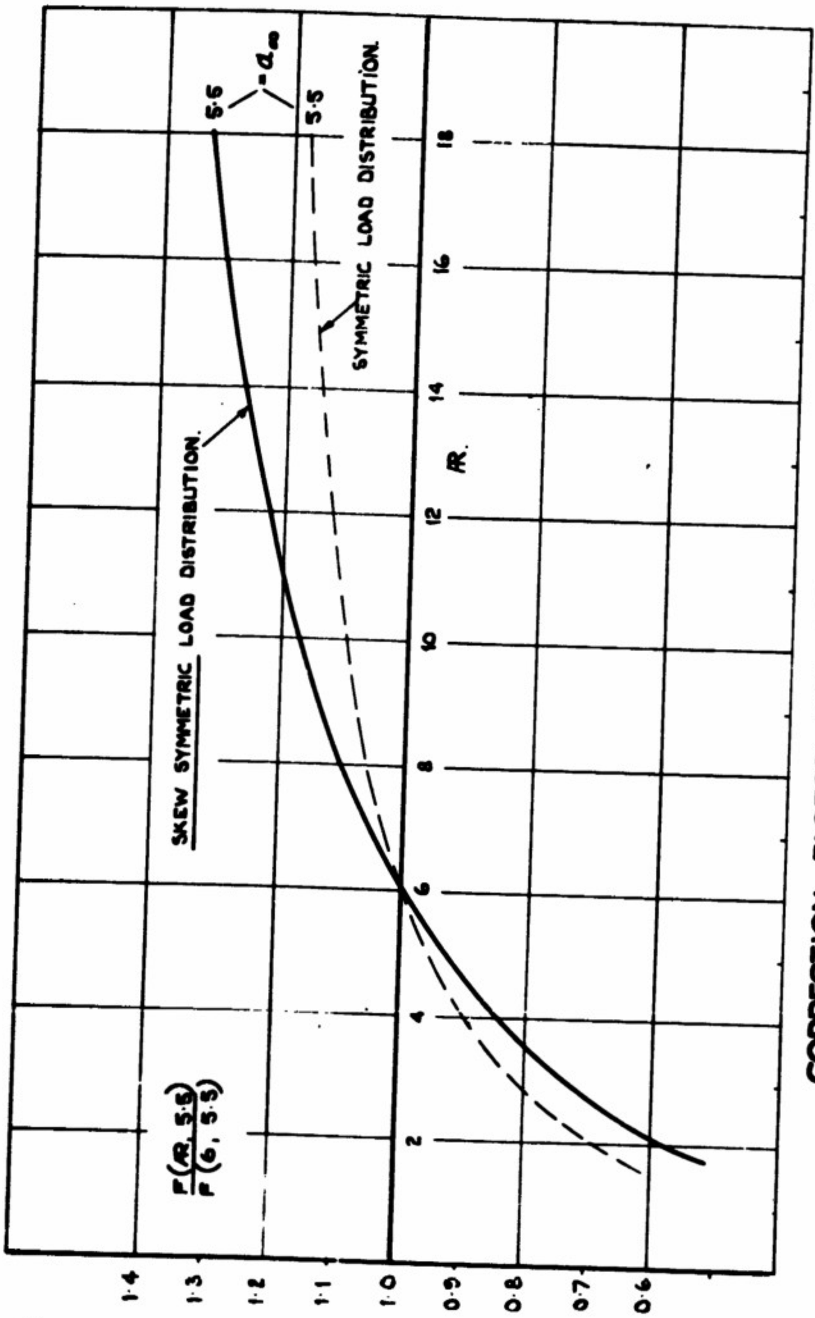
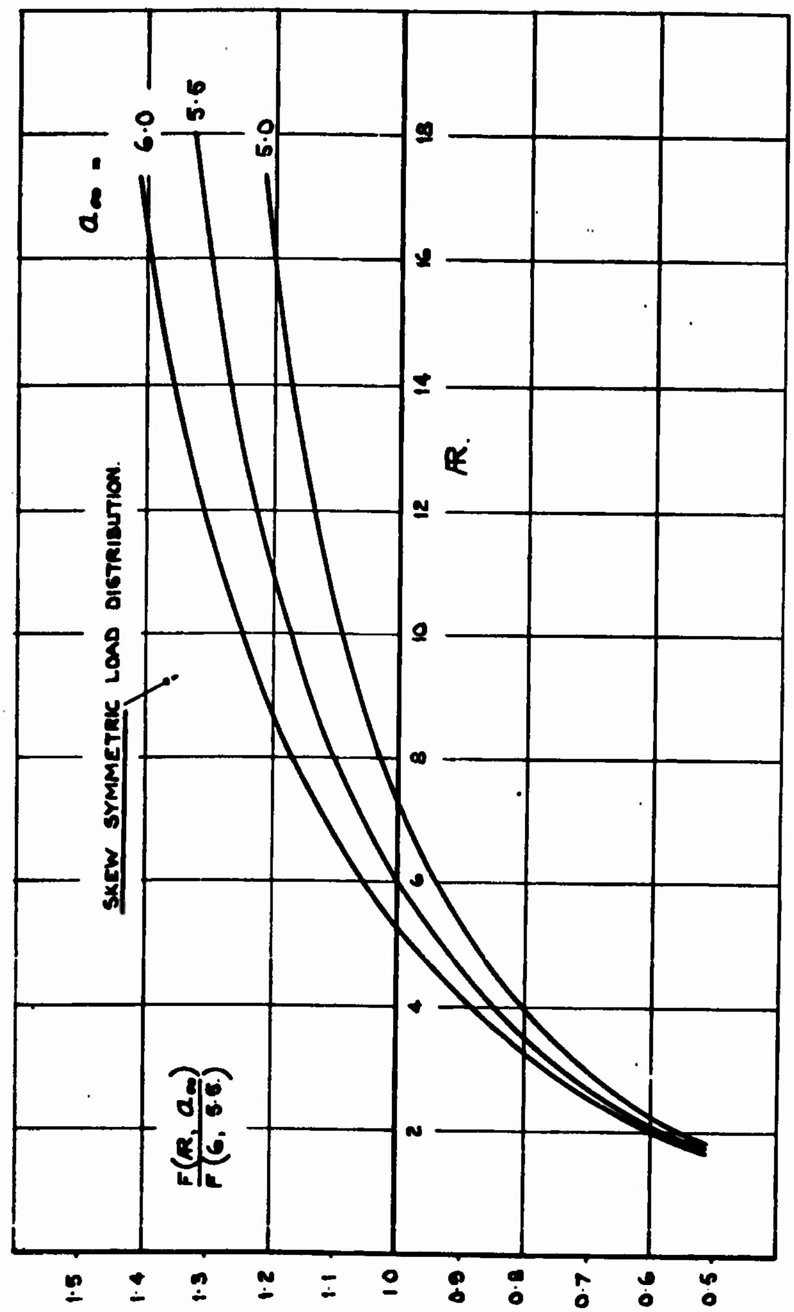
FIG. 6a



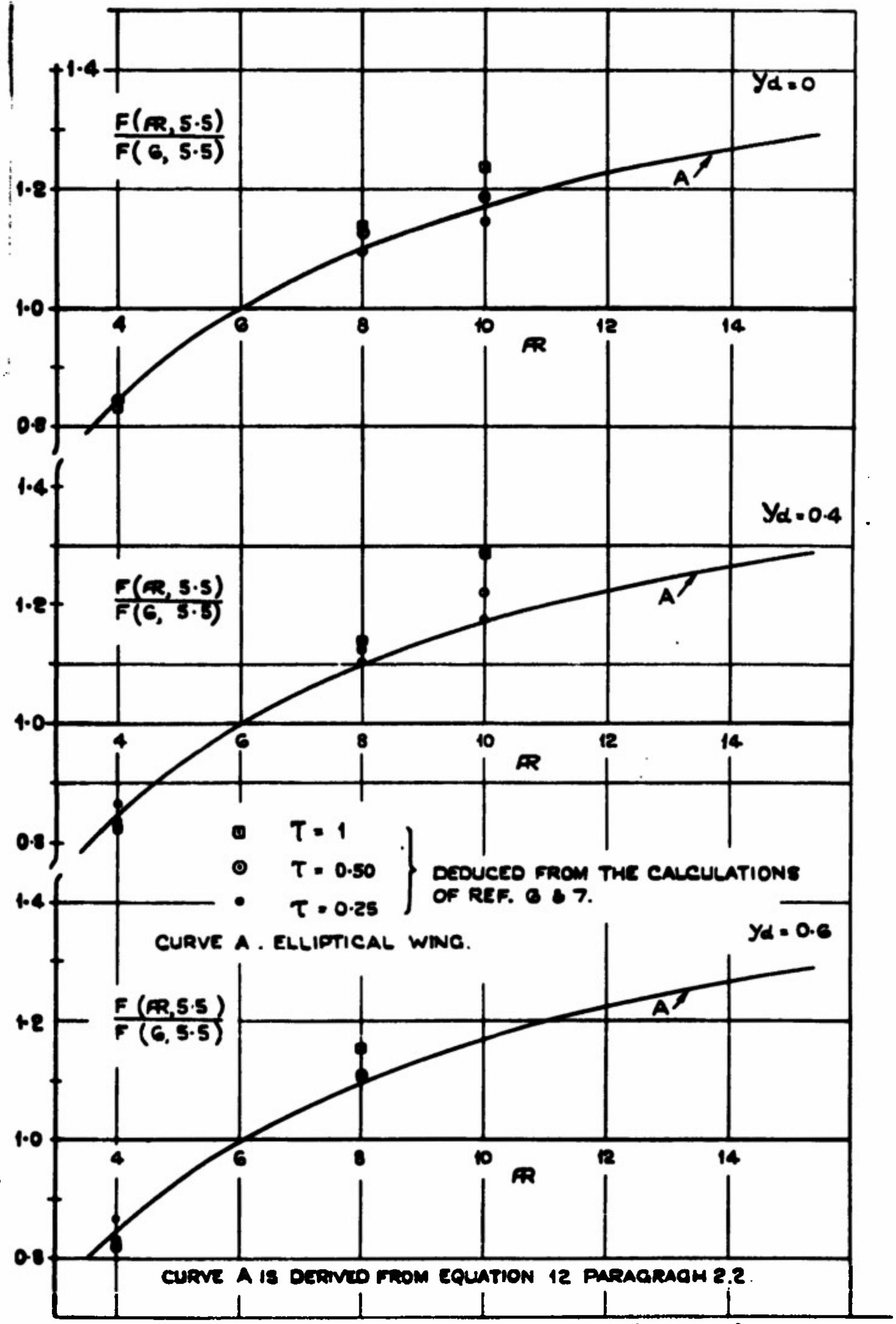
CORRECTION FACTOR FOR ASPECT RATIO, $\frac{F(A\!R,\ 5{\cdot}5)}{F(6,\ 5{\cdot}5)}$, FOR SKEW SYMMETRIC LOAD DISTRIBUTION (ELLIPTIC WING)

FIG. 6b



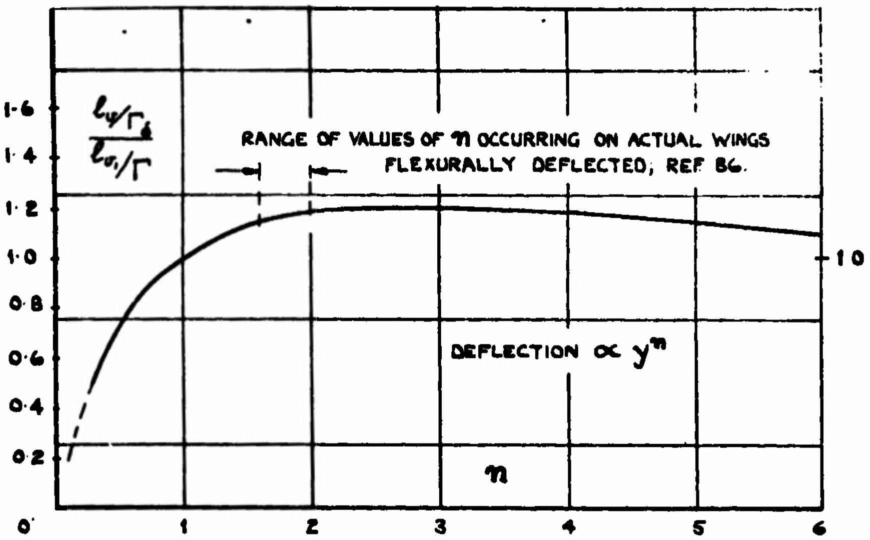
CORRECTION FACTOR FOR ASPECT RATIO $\frac{F(A\!\!R,\ a_\infty)}{F(6,\ 5{\cdot}5)}$, FOR SKEW SYMMETRIC LOAD DISTRIBUTION (ELLIPTIC WING.)

FIG. 7.



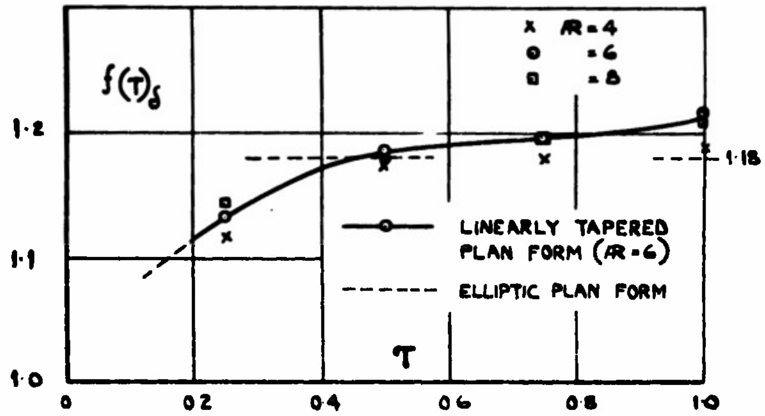
CORRECTION FACTOR FOR ASPECT RATIO $\frac{F(AR,\ a_{00})}{F(6,\ 5\cdot5)}$, FOR SKEW SYMMETRIC LOAD DISTRIBUTION ON ELLIPTIC AND LINEARLY TAPERED WINGS.

FIG 8



CHANGES IN l_v/Γ_δ WITH n EXPRESSED AS A FRACTION OF l_{v_1}/Γ.

FIG. 9



CALCULATED TAPER EFFECT ON l_v DUE TO FLEXURE FOR A WING WITH PARABOLIC DEFLECTION. (i.e. $n = 2$)

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